

# Electroweak Chiral Lagrangian from Natural Topcolor-assisted Technicolor Model

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Based on previous studies computing coefficients of the electroweak chiral Lagrangian from C.T.Hill's schematic topcolor-assisted technicolor model, we generalize the calculation to K.Lane's prototype natural topcolor-assisted technicolor model. We find that typical features of the model are qualitatively similar as those of Hill's model, but Lane's model prefers smaller technicolor group and  $Z'$  mass must be smaller than 400GeV, further  $S$  parameter is around order of +1 mainly due to existence of three doublets of techniquarks. We obtain the values for all coefficients of the electroweak chiral Lagrangian up to order of  $p^4$ . Apart from negative large four fermion coupling values, ETC impacts on the electroweak chiral Lagrangian coefficients are small, since techniquark self energy which determines these coefficients in general receives almost no influence from ETC induced four fermion interactions except for its large momentum tail.

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## I. INTRODUCTION

Topcolor assisted technicolor (TC2) model realizes the electroweak symmetry breaking (EWSB) by joining technicolor (TC) and topcolor together to remove the objections that topcolor is unnatural and that TC cannot generate a large top mass. In the first schematic model proposed by C.T.Hill[1], EWSB is driven mainly by TC interactions and light quark and lepton masses are expected to be generated by extended technicolor (ETC). The third generation  $(t, b)_{L,R}$  is arranged to transform with the usual quantum numbers under the gauge group  $SU(3)_1 \otimes U(1)_1$  while  $(u, d), (c, s)$  transform under a separate group  $SU(3)_2 \otimes U(1)_2$ . At a scale of order 1TeV,  $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2$  is dynamically broken to the diagonal subgroup  $SU(3)_C \otimes U(1)_Y$ , and  $SU(3)_1 \otimes U(1)_1$  interactions are supercritical for  $t$  quark leading top condensation, but subcritical for  $b$  quark causing no bottom condensation which achieve large mass difference between  $t$  and  $b$  quarks.

As a candidate of new physics model, before any new particles such as  $Z'$  or colorons predicted in TC2 model show up in upcoming collider experiments, behavior of the model in low energy region for those discovered particles can be tested and described by its effective electroweak chiral Lagrangian (EWCL) [2, 3] which, as a model independent platform of investigating EWSB mechanism, parameterizes the model by a set of coefficients. Starting from this EWCL, except phenomenological research on fixing the coefficients of EWCL from experiments data, theoretical studies concentrate on com-

puting the values of the coefficients from the detail underlying model. Considering that TC2 model involves strongly coupled dynamics for which traditional perturbative expansion fails in the computation for the coefficients of EWCL, in previous paper[4], we built up a formulation computing bosonic part of EWCL coefficients up to order of  $p^4$  for one-doublet TC model[5] and Hill's schematic model[1]. This formulation is of general purposes, it can be applied to many other strongly coupled models. Then EWCL becomes an universal platform on which we can compare different underlying models with experiment data and extract out the true physical theory of our real world. To achieve the aim of this comparison, the left theoretical works are to compute EWCL coefficients model by models. Present work is the second paper starting from Ref.[4] for series computations for various strongly coupled new physics models. Here we focus on K.Lane's prototype natural TC2 model[6].

In original Hill's model, effects of ETC interactions are only qualitatively estimated, effective four fermion interactions induced by ETC (EFFIETC) are even not explicitly written down in Ref.[1]. Accordingly our previous computations [4] also do not involve possible ETC's contributions. Examining ETC effects, Chivukula, Dobrescu and Terning (CDT)[7] argued that the TC2 proposal cannot be both natural and consistent with experimental measurements of the parameter  $\rho = M_W^2/M_Z^2 \cos^2 \theta_W$ . In extreme case, even for degenerate up and down type technifermions of third generation are likely to have custodial-isospin violating couplings to the strong  $U(1)_1$  since part of  $m_t$  must arise from ETC, and this leads to large contributions to  $\rho$  parameter which contradicts with experiment data.<sup>1</sup> To overcome this dif-

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<sup>1</sup> In fact, the detail up and down type technifermions of third generation are formally arranged not participating  $U(1)_1$  interaction by vanishing their  $U(1)_1$  charges in original Hill's model and then

ficulty, instead of conventional one doublet third generation technifermions, K.Lane and E.Eichten propose their model[6] by introducing two sets of technifermion doublets for third generation techniquarks with different  $U(1)_1$  charges but up and down type technifermions in the same doublet possessing the same  $U(1)_1$  charges:  $T_{L,R}^t = (U^t, D^t)_{L,R}$  giving the top quark its ETC-mass;  $T_{L,R}^b = (U^b, D^b)_{L,R}$  giving the bottom quark its ETC-mass, these cut the intimate relation between custodial-isospin violation from techniquarks and t-b mass difference. Due to this important role of ETC interactions in Lane's model, its effects in EWCL is worth of examination and this paper is not only for computing EWCL coefficients of Lane's model, but also for investigating ETC effects on these coefficients.

In next section, we apply our formulation developed in Ref.[4] to Lane's model[6]. We perform dynamical calculations through several steps: first integrate in goldstone field  $U$ , then integrate out technigluons and techniquarks by solving Schwinger-Dyson equation (SDE) for techniquarks and compute result effective action, further integrating out  $Z'$  and finally obtain EWCL coefficients. Section III is the discussion. In the appendix, we list some requisite formulae.

## II. DERIVATION OF EWCL FROM LANE'S MODEL

Consider prototype natural TC2 model proposed by K.Lane and E.Eichten[6]. The TC group is not specified in Ref.[6], but chosen to be  $SU(N)$  in later Lane's improved model[8]. For definiteness, we take  $G_{TC} = SU(N)$ . The gauge charge assignments of techniquarks in  $G_{TC} \otimes SU(3)_1 \otimes SU(3)_2 \otimes SU(2)_L \otimes U(1)_{Y_1} \otimes U(1)_{Y_2}$  are shown as Table I for which we choose the case B solution<sup>2</sup> to the anomaly conditions of Ref.[6].

The action of the symmetry breaking sector then is

$$S_{\text{SBS}}[G_{\mu\nu}^\alpha A_{1\mu\nu}^A A_{2\mu\nu}^A W_{\mu\nu}^a B_{1\mu\nu} B_{2\mu\nu} \bar{T}^l, T^l, \bar{T}^t, T^t, \bar{T}^b, T^b] \\ = \int d^4x (\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{techniquark}} + \mathcal{L}_{\text{breaking}} + \mathcal{L}_{4T}), \quad (1)$$

do not cause large contribution to  $\rho$ . This result is compatible with that obtained in Ref.[4]. But this naive arrangement is not realistic in the sense, as mentioned by CDT[7], that to give top and bottom (which must have different  $U(1)_1$  charges to allow for their different masses) ETC masses, the different right-handed technifermions to which top and bottom quarks couple must have different  $U(1)_1$  charges.

<sup>2</sup> Case A solution, as mentioned by K.Lane in Ref.[6], would not be possible to generate proper ETC masses for the t and b quarks and therefore not considered in this work.

**TABLE I.** Gauge charge assignments of techniquarks for prototype natural TC2 model given in Ref.[6]. These techniquarks are  $SU(3)_1 \otimes SU(3)_2$  singlets.

field	$SU(N)$	$SU(2)_L$	$U(1)_{Y_1}$	$U(1)_{Y_2}$
$T_L^l$	N	2	0	0
$U_R^l$	N	1	0	$\frac{1}{2}$
$D_R^l$	N	1	0	$-\frac{1}{2}$
$T_L^t$	N	2	-1	1
$U_R^t$	N	1	$-\frac{1}{2}$	1
$D_R^t$	N	1	$-\frac{1}{2}$	0
$T_L^b$	N	2	1	-1
$U_R^b$	N	1	$\frac{1}{2}$	0
$D_R^b$	N	1	$\frac{1}{2}$	-1

with different part of Lagrangian given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}^\alpha F^{\alpha,\mu\nu} - \frac{1}{4}A_{1\mu\nu}^A A_1^{A\mu\nu} - \frac{1}{4}A_{2\mu\nu}^A A_2^{A\mu\nu} \\ - \frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{1\mu\nu} B^{1,\mu\nu} - \frac{1}{4}B_{2\mu\nu} B_2^{\mu\nu}, \quad (2)$$

$$\mathcal{L}_{\text{techniquark}} \quad (3)$$

$$= \bar{T}^l(i\not{\partial} - g_{TC}t^\alpha \not{G}^\alpha - g_2\frac{\tau^a}{2}\not{W}^a P_L - \frac{1}{2}q_2\not{B}_2\tau^3 P_R)T^l \\ + \bar{T}^t(i\not{\partial} - g_{TC}t^\alpha \not{G}^\alpha - g_2\frac{\tau^a}{2}\not{W}^a P_L + q_1\not{B}_1 P_L \\ - q_2\not{B}_2 P_L + \frac{1}{2}q_1\not{B}_1 P_R - (\frac{1}{2} + \frac{\tau^3}{2})q_2\not{B}_2 P_R)T^t \\ + \bar{T}^b(i\not{\partial} - g_{TC}t^\alpha \not{G}^\alpha - g_2\frac{\tau^a}{2}\not{W}^a P_L - q_1\not{B}_1 P_L \\ + q_2\not{B}_2 P_L - \frac{1}{2}q_1\not{B}_1 P_R + (\frac{1}{2} - \frac{\tau^3}{2})q_2\not{B}_2 P_R)T^b, \quad (4)$$

$$\mathcal{L}_{4T} = \mathcal{H}_{\text{diag}}, \quad (4)$$

$$\mathcal{H}_{\text{diag}} = \frac{g_{\text{ETC}}^2}{M_{\text{ETC}}^2} \bar{T}_L^i \gamma^\mu T_L^i (b_U \bar{U}_R^j \gamma_\mu U_R^j + b_D \bar{D}_R^j \gamma_\mu D_R^j), \quad (5)$$

where  $g_{TC}$ ,  $g_2$ ,  $q_1$  and  $q_2$  are the coupling constants of, respectively,  $SU(N)$ ,  $SU(2)_L$ ,  $U(1)_{Y_1}$  and  $U(1)_{Y_2}$  (since techniquarks are  $SU(3)_1 \otimes SU(3)_2$  singlets, corresponding coupling constants do not show up here); and the corresponding gauge fields (field strength tensors) are denoted by  $G_\mu^\alpha$ ,  $W_\mu^a$ ,  $B_{1\mu}$  and  $B_{2\mu}$  ( $F_{\mu\nu}^\alpha$ ,  $W_{\mu\nu}^a$ ,  $B_{1\mu\nu}$  and  $B_{2\mu\nu}$ ) with the superscript  $\alpha$  runs from 1 to  $N^2 - 1$  and  $a$  from 1 to 3 ( $SU(3)_1 \otimes SU(3)_2$  gauge fields and field strength tensors are denoted by  $A_{1\mu}^A$ ,  $A_{2\mu}^A$  and  $A_{1\mu\nu}^A$ ,  $A_{2\mu\nu}^A$  with the superscript  $A$  runs from 1 to 8);  $t^\alpha = \lambda^\alpha/2$  ( $\alpha = 1, \dots, N^2 - 1$ ) and  $\tau^a$  ( $a = 1, 2, 3$ ) are, respectively, Gell-Mann and Pauli matrices.  $P_L = (1 \pm \gamma_5)/2$ .

Ordinary quarks are neglected, since we only discuss bosonic part of EWCL.<sup>3</sup> For ETC induced four fermion interactions  $\mathcal{L}_{4T}$ , although in original Ref.[6], except  $\mathcal{H}_{\text{diag}}$ , there other different kinds of interactions, such as  $\mathcal{H}_{l\bar{t}t\bar{b}}$  and  $\mathcal{H}_{l\bar{b}b\bar{t}}$ , consider these non-diagonal interactions will induce non-diagonal condensates which violate the preferred requirement  $\langle \bar{U}_L^i U_R^j \rangle = \langle \bar{D}_L^i D_R^j \rangle \propto \delta_{ij}$  for  $i, j = l, t, b$  given in Ref.[6], we drop them in our calculation.

In Ref.[6], an operator effecting  $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2$  breaking to  $SU(3)_C \otimes U(1)_Y$  is needed. We introduce a  $3 \times 3$  matrix scalar field  $\Phi$  to take the role of this operator to break  $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2}$  to  $SU(3)_C \otimes U(1)_Y$  which leads massive colorons and  $Z'$ . This scalar field transforms as  $(\bar{3}, 3, \frac{5}{6}, -\frac{5}{6})$  under the group  $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2}$  which leads covariant derivative

$$D_\mu \Phi = \partial_\mu \Phi + i\Phi(h_1 \frac{\lambda^{A*}}{2} A_{1\mu}^A - \frac{5}{6} q_1 B_{1\mu}) - i(h_2 \frac{\lambda^A}{2} A_{2\mu}^A - \frac{5}{6} q_2 B_{2\mu})\Phi,$$

with  $h_1$  and  $h_2$  are the coupling constants of  $SU(3)_1 \otimes SU(3)_2$  and corresponding Lagrangian can be written as

$$\mathcal{L}_H = \frac{1}{2} \text{tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] + V(\Phi) \quad (6)$$

in which potential  $V(\Phi)$  is assumed to cause vacuum condensate  $\Phi_{ij} = v\delta_{ij}$  and the leading effects can be obtained by just replacing  $\Phi$  with its vacuum expectation value in (6),

$$\mathcal{L}_{H=vv} = \frac{1}{4} \frac{g_3^2}{\sin^2 \theta \cos^2 \theta} B_\mu^A B^{A\mu} + \frac{25}{72} \frac{g_1^2}{\sin^2 \theta' \cos^2 \theta'} Z'_\mu Z'^\mu, \quad (7)$$

where the SM  $U(1)_Y$  field  $B_\mu$  with generator  $Y = Y_1 + Y_2$  and the  $U(1)'$  field  $Z'_\mu$  (the gluon  $A_\mu^A$  and coloron  $B_\mu^A$ ) are defined by orthogonal rotations with mixing angle  $\theta'$  ( $\theta$ ):

$$\begin{pmatrix} B_{1\mu} & B_{2\mu} \end{pmatrix} = \begin{pmatrix} Z'_\mu & B_\mu \end{pmatrix} \begin{pmatrix} \cos \theta' & -\sin \theta' \\ \sin \theta' & \cos \theta' \end{pmatrix}, \quad (8a)$$

$$\begin{pmatrix} A_{1\mu}^A & A_{2\mu}^A \end{pmatrix} = \begin{pmatrix} B_\mu^A & A_\mu^A \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}, \quad (8b)$$

with

$$g_1 \equiv q_1 \sin \theta' = q_2 \cos \theta' \quad g_3 \equiv h_1 \sin \theta = h_2 \cos \theta. \quad (9)$$

<sup>3</sup> For top quark, its effect should be considered due to its large mass comparable to symmetry breaking scale. There is an EFFIETC  $\mathcal{H}_{t\bar{t}} = \frac{g_{\text{ETC}}^2}{M_{\text{ETC}}^2} \bar{t}_L \gamma^\mu U_L^t \bar{U}_R^t \gamma_\mu t_R + h.c.$  responsible for top mass. This interaction should be included in our calculation in principle and if top quark has nonzero condensate, this interaction will contribute to techniquark self energy. Since Ref.[6] treats this term as a perturbation, we can ignore it at leading order of our coefficients computations.

The coloron field  $B_\mu^A$  does not couple to other fields except to ordinary fermions at present order of approximation, so we can ignore their contributions to bosonic part of EWCL.<sup>4</sup> i.e. we can take

$$\mathcal{L}_{\text{breaking}} = \frac{1}{2} M_0^2 Z'_\mu Z'^\mu \quad M_0^2 = \frac{25}{36} \frac{g_1^2 v^2}{\sin^2 \theta' \cos^2 \theta'}. \quad (10)$$

With above preparations, the strategy to derive the EWCL from Lane's model can be formulated as

$$\exp \left( i S_{\text{EW}}[W_\mu^a, B_\mu] \right) \quad (11)$$

$$= \int \mathcal{D}\bar{T}^l \mathcal{D}T^l \mathcal{D}\bar{T}^t \mathcal{D}T^t \mathcal{D}\bar{T}^b \mathcal{D}T^b \mathcal{D}G_\mu^\alpha \mathcal{D}Z'_\mu \exp \left[ i \times S_{\text{SBS}}[G_\mu^\alpha, 0, W_\mu^a, B_{1\mu}, B_{2\mu}, \bar{T}^l, T^l, \bar{T}^t, T^t, \bar{T}^b, T^b] \right] \\ = \mathcal{N}[W_\mu^a, B_\mu] \int \mathcal{D}\mu(U) \exp \left( i S_{\text{eff}}[U, W_\mu^a, B_\mu] \right), \quad (12)$$

where  $A_\mu^A$  related to  $A_{1\mu}^A$  and  $A_{2\mu}^A$  through (8b) is ordinary gluon field,  $U(x)$  is a dimensionless unitary unitary modular matrix field in EWCL, and  $\mathcal{D}\mu(U)$  denotes normalized functional integration measure on  $U$ . The normalization factor  $\mathcal{N}[W_\mu^a, B_\mu]$  is determined through requirement that when the TC and ETC interactions are switched off,  $S_{\text{eff}}[U, W_\mu^a, B_\mu]$  must vanishes. This leads following electroweak gauge fields  $W_\mu^a$ ,  $B_\mu$  dependent  $\mathcal{N}[W_\mu^a, B_\mu]$ ,

$$\mathcal{N}[W_\mu^a, B_\mu] = \int \mathcal{D}\bar{T}^l \mathcal{D}T^l \mathcal{D}\bar{T}^t \mathcal{D}T^t \mathcal{D}\bar{T}^b \mathcal{D}T^b \mathcal{D}G_\mu^\alpha \mathcal{D}Z'_\mu \\ \times e^{i S_{\text{SBS}}|_{\text{ignore TC,ETC}, A_{1\mu}^A=A_{2\mu}^A=0}}. \quad (13)$$

Since there are many steps in deriving EWCL, in following several subsections, we discuss them separately.

### A. Integrating in Goldstone Field $U$

In terms of  $Z'$  and  $B$  fields given by (8a), we can rewrite techniquark interaction (3) as

$$\mathcal{L}_{\text{techniquark}} = \bar{\psi}(i\not{D} - g_{\text{TC}} t^\alpha \not{G}^\alpha + \not{V} + \not{A} \gamma^5) \psi, \quad (14)$$

where all three doublets techniquarks are arranged in one by six matrix  $\psi = (U^l, D^l, U^t, D^t, U^b, D^b)^T$  and

$$V_\mu = \left( -\frac{1}{2} g_2 \frac{\tau^a}{2} W_\mu^a - \frac{1}{2} g_1 \frac{\tau^3}{2} B_\mu \right) \otimes \mathbf{I} + Z_{V\mu}, \quad (15)$$

$$A_\mu = \left( \frac{1}{2} g_2 \frac{\tau^a}{2} W_\mu^a - \frac{1}{2} g_1 \frac{\tau^3}{2} B_\mu \right) \otimes \mathbf{I} + Z_{A\mu}, \quad (16)$$

<sup>4</sup> One can consider higher order corrections by including in (7) the quantum fluctuation effects of field  $\Phi$ . Since these effects depend on detail of symmetry breaking mechanism which is not specified in Ref.[6], in order not to deviate original Lane's model too much, we ignore them in present paper.

with  $\mathbf{I} = \text{diag}(1, 1, 1)$ ,  $Z_{V\mu} = \text{diag}(Z_{V\mu}^l, Z_{V\mu}^t, Z_{V\mu}^b)$ ,  $Z_{A\mu} = \text{diag}(Z_{A\mu}^l, Z_{A\mu}^t, Z_{A\mu}^b)$  and

$$Z_{V\mu}^l = \frac{1}{4}g_1 \tan \theta' Z_{\mu}' \tau^3, \quad (17)$$

$$\begin{aligned} Z_{V\mu}^t &= g_1 Z_{\mu}' \left[ -\frac{3}{4} \cot \theta' + \left( \frac{3}{4} + \frac{1}{4} \tau^3 \right) \tan \theta' \right], \\ Z_{V\mu}^b &= g_1 Z_{\mu}' \left[ -\frac{3}{4} \cot \theta' - \left( \frac{3}{4} - \frac{1}{4} \tau^3 \right) \tan \theta' \right], \\ Z_{A\mu}^l &= \frac{1}{4}g_1 \tan \theta' Z_{\mu}' \tau^3, \\ Z_{A\mu}^t &= g_1 Z_{\mu}' \left[ -\frac{1}{4} \cot \theta' + \left( -\frac{1}{4} + \frac{1}{4} \tau^3 \right) \tan \theta' \right], \\ Z_{A\mu}^b &= g_1 Z_{\mu}' \left[ \frac{1}{4} \cot \theta' + \left( \frac{1}{4} + \frac{1}{4} \tau^3 \right) \tan \theta' \right]. \end{aligned} \quad (18)$$

The Lagrangian (1) is locally  $SU(2)_L \times U(1)_Y$  invariant and approximately globally  $SU(6)_L \times SU(6)_R$  invariant. We introduce a local  $2 \times 2$  operator  $O(x)$  as  $O(x) \equiv \text{tr}_{lc}[T_L^l(x)\bar{T}_R^l(x) + T_L^t(x)\bar{T}_R^t(x) + T_L^b(x)\bar{T}_R^b(x)]$  with  $\text{tr}_{lc}$  is the trace with respect to Lorentz and TC indices. The transformation of  $O(x)$  under  $SU(2)_L \times U(1)_Y$  is  $O(x) \rightarrow V_L(x)O(x)V_R^\dagger(x)$  (with  $V_L = e^{i\frac{\tau^a}{2}\theta^a}$  and  $V_R = e^{i\frac{\tau^a}{2}\theta^0}$ ). Then we decompose  $O(x)$  as  $O(x) = \xi_L^\dagger(x)\sigma(x)\xi_R(x)$  with the  $\sigma(x)$  represented by a hermitian matrix describes the modular degree of freedom; while  $\xi_L(x)$  and  $\xi_R(x)$  are represented by unitary matrices describe the phase degree of freedom of  $SU(2)_L$  and  $U(1)_Y$  respectively. Now we define a new field  $U(x)$  as  $U(x) \equiv \xi_L^\dagger(x)\xi_R(x)$  which is the nonlinear realization of the goldstone boson field in EWCL. Subtracting the  $\sigma(x)$  field, we find that the present decomposition results in a constraint  $\xi_L(x)O(x)\xi_R^\dagger(x) - \xi_R(x)O^\dagger(x)\xi_L^\dagger(x) = 0$ , the functional expression of it is

$$\int \mathcal{D}\mu(U) \mathcal{F}[O] \delta(\xi_L O \xi_R^\dagger - \xi_R O^\dagger \xi_L^\dagger) = \text{const.}, \quad (19)$$

where  $\mathcal{D}\mu(U)$  is an effective invariant integration measure;  $\mathcal{F}[O]$  only depends on  $O$ . Substituting identity (19) into (12), we obtain

$$\begin{aligned} & \int \mathcal{D}G_\mu^\alpha \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}Z'_\mu \exp \left( iS_{\text{SBS}}|_{A_{1\mu}^A = A_{2\mu}^A = 0} \right) \\ &= \int \mathcal{D}\mu(U) \mathcal{D}Z'_\mu \exp \left( iS_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu] \right), \end{aligned} \quad (20)$$

where  $\mathcal{D}\bar{\psi} \mathcal{D}\psi$  is the shorthand notation for  $\mathcal{D}\bar{T}^l \mathcal{D}T^l \mathcal{D}\bar{T}^t \mathcal{D}T^t \mathcal{D}\bar{T}^b \mathcal{D}T^b$  and

$$\begin{aligned} & S_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu] \\ &= \int d^4x \left( -\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu} Z'^{\mu\nu} \right. \\ & \quad \left. + \frac{1}{2}M_0^2 Z'_\mu Z'^\mu \right) - i \log \int \mathcal{D}G_\mu^\alpha \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{F}[O] \\ & \quad \times \delta(\xi_L O \xi_R^\dagger - \xi_R O^\dagger \xi_L^\dagger) \exp \left\{ i \int d^4x \left[ -\frac{1}{4}F_{\mu\nu}^\alpha F^{\alpha,\mu\nu} \right. \right. \end{aligned} \quad (21)$$

$$\left. \left. + \bar{\psi}(i\bar{\not{D}} - g_{\text{TC}} t^\alpha \not{G}^\alpha + \not{V} + \not{A}\gamma^5)\psi + \mathcal{L}_{4T} \right] \right\}.$$

From (12),  $S_{\text{eff}}$  relates to  $S_{Z'}$  by

$$\mathcal{N}[W_\mu^a, B_\mu] e^{iS_{\text{eff}}[U, W_\mu^a, B_\mu]} = \int \mathcal{D}Z'_\mu e^{iS_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu]} \quad (22)$$

To match the correct normalization, we introduce in the argument of logarithm function the normalization factor  $\int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{i \int d^4x \bar{\psi}(i\bar{\not{D}} + \not{V} + \not{A}\gamma^5)\psi} = \exp \text{Tr} \log(i\bar{\not{D}} + \not{V} + \not{A}\gamma^5)$  and then take a special  $SU(2)_L \times U(1)_Y$  rotation, as  $V_L(x) = \xi_L(x)$  and  $V_R(x) = \xi_R(x)$ , on both numerator and denominator of the normalization factor

$$\begin{aligned} & S_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu] \\ &= \int d^4x \left( -\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu} Z'^{\mu\nu} \right. \\ & \quad \left. + \frac{1}{2}M_0^2 Z'_\mu Z'^\mu \right) - i \text{Tr} \log(i\bar{\not{D}} + \not{V} + \not{A}\gamma^5) \\ & \quad - i \log \frac{\int \mathcal{D}G_\mu^\alpha \mathcal{D}\bar{\psi}_\xi \mathcal{D}\psi_\xi \mathcal{F}[O_\xi] \delta(O_\xi - O_\xi^\dagger) e^{iS'}}{\int \mathcal{D}\bar{\psi}_\xi \mathcal{D}\psi_\xi e^{iS'}|_{\text{ignore TC,ETC}}} \end{aligned} \quad (23)$$

$$\begin{aligned} S' &= \int d^4x \left[ -\frac{1}{4}F_{\mu\nu}^\alpha F^{\alpha,\mu\nu} + \bar{\psi}_\xi(i\bar{\not{D}} - g_{\text{TC}} t^\alpha \not{G}^\alpha \right. \\ & \quad \left. + \not{V}_\xi + \not{A}_\xi \gamma^5)\psi_\xi + \mathcal{L}_{\xi 4T} \right], \end{aligned} \quad (24)$$

where the rotated fields are denoted by subscript  $\xi$  and they are defined as follows

$$\begin{aligned} T_\xi^i &= P_L \xi_L(x) T_L^i(x) + P_R \xi_R(x) T_R^i(x), \quad i = l, t, b \\ O_\xi(x) &\equiv \xi_L(x) O(x) \xi_R^\dagger(x) \quad Z'_{\xi,\mu}(x) \equiv Z'_\mu(x), \end{aligned} \quad (25)$$

$$g_2 \frac{\tau^a}{2} W_{\xi,\mu}^a(x) \equiv \xi_L(x) [g_2 \frac{\tau^a}{2} W_\mu^a(x) - i\partial_\mu] \xi_L^\dagger(x) \quad (26)$$

$$g_1 \frac{\tau^3}{2} B_{\xi,\mu}(x) \equiv \xi_R(x) [g_1 \frac{\tau^3}{2} B_\mu(x) - i\partial_\mu] \xi_R^\dagger(x). \quad (27)$$

and  $\mathcal{L}_{\xi 4T}$  is  $\mathcal{L}_{4T}$  with TC fields replaced with rotated ones. It can be shown that

$$\mathcal{L}_{\xi 4T} = \mathcal{L}_{4T}. \quad (28)$$

Action (23) can be further decomposed as

$$\begin{aligned} & S_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu] \\ &= \int d^4x \left( -\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu} Z'^{\mu\nu} \right. \\ & \quad \left. + \frac{1}{2}M_0^2 Z'_\mu Z'^\mu \right) + S_{\text{norm}}[U, W_\mu^a, B_\mu, Z'_\mu] \\ & \quad + S_{\text{anom}}[U, W_\mu^a, B_\mu, Z'_\mu], \end{aligned} \quad (29)$$

where

$$\begin{aligned} & S_{\text{norm}}[U, W_\mu^a, B_\mu] \\ &= -i \log \int \mathcal{D}G_\mu^\alpha \mathcal{D}\bar{\psi}_\xi \mathcal{D}\psi_\xi \mathcal{F}[O_\xi] \delta(O_\xi - O_\xi^\dagger) e^{iS'}, \end{aligned} \quad (30)$$

and

$$iS_{\text{anom}}[U, W_\mu^a, B_\mu, Z'_\mu] = \text{Tr} \log(i\bar{\psi} + \bar{V} + \bar{A}\gamma^5) - \text{Tr} \log(i\bar{\psi} + \bar{V}_\xi + \bar{A}_\xi\gamma^5). \quad (31)$$

The transformations of the rotated fields under  $SU(2)_L \times U(1)_Y$  are  $\psi_\xi(x) \rightarrow h(x)\psi_\xi(x)$ ,  $O_\xi(x) \rightarrow h(x)O_\xi(x)h^\dagger(x)$  with  $h(x)$  describes a hidden local  $U(1)$  symmetry. Thus, the chiral symmetry  $SU(2)_L \otimes U(1)_Y$  covariance of the unrotated fields has been transferred totally to the hidden symmetry  $U(1)$  covariance of the rotated fields.

## B. Integrating out technigluons and techniquarks

With technique developed in Ref.[4], the integration over technigluon fields in Eq.(30) can be formally integrated out with help of full  $n$ -point Green's function of the  $G_\mu^{\alpha_1 \dots \alpha_n}$ -field  $G_{\mu_1 \dots \mu_n}^{\alpha_1 \dots \alpha_n}$ ,

$$e^{iS_{\text{norm}}[U, W_\mu^a, B_\mu, Z'_\mu]} = \int \mathcal{D}\bar{\psi}_\xi \mathcal{D}\psi_\xi \mathcal{F}[O_\xi] \delta(O_\xi - O_\xi^\dagger) \exp \left\{ i \int d^4x [\bar{\psi}_\xi(i\bar{\not{D}} + \bar{V}_\xi + \bar{A}_\xi\gamma^5)\psi_\xi + \mathcal{L}_{\xi 4T}] + \sum_{n=2}^{\infty} \int d^4x_1 \dots d^4x_n \frac{(-ig_{\text{TC}})^n}{n!} \times G_{\mu_1 \dots \mu_n}^{\alpha_1 \dots \alpha_n}(x_1, \dots, x_n) J_{\xi, \alpha_1}^{\mu_1}(x_1) \dots J_{\xi, \alpha_n}^{\mu_n}(x_n) \right\}, \quad (32)$$

where effective source  $J_\xi^{\alpha\mu}(x)$  is identified as  $J_\xi^{\alpha\mu}(x) \equiv \bar{\psi}_\xi(x) t^\alpha \gamma^\mu \psi_\xi(x)$ .

### 1. Schwinger-Dyson Equation for Techniquark Propagator

To show that the TC interaction indeed induces the condensate  $\langle \bar{\psi}\psi \rangle \neq 0$  which triggers EWSB, we explicitly calculate the behavior of techniquark propagator  $S^{\sigma\rho}(x, x') \equiv \langle \psi_\xi^\sigma(x) \bar{\psi}_\xi^\rho(x') \rangle$  in the following. *Neglecting the factor  $\mathcal{F}[O_\xi] \delta(O_\xi - O_\xi^\dagger)$* , the total functional derivative of the integrand with respect to  $\bar{\psi}_\xi^\sigma(x)$  is zero,

$$0 = \int \mathcal{D}\bar{\psi}_\xi \mathcal{D}\psi_\xi \frac{\delta}{\delta \bar{\psi}_\xi^\sigma(x)} \exp \left[ \int d^4x (\bar{\psi}_\xi I + \bar{I} \psi_\xi) + i \int d^4x [\bar{\psi}_\xi(i\bar{\not{D}} + \bar{V}_\xi + \bar{A}_\xi\gamma^5)\psi_\xi + \mathcal{L}_{\xi 4T}] + \sum_{n=2}^{\infty} \int d^4x_1 \dots d^4x_n \frac{(-ig_{\text{TC}})^n}{n!} G_{\mu_1 \dots \mu_n}^{\alpha_1 \dots \alpha_n}(x_1, \dots, x_n) J_{\xi, \alpha_1}^{\mu_1}(x_1) \dots J_{\xi, \alpha_n}^{\mu_n}(x_n) \right],$$

where  $I(x)$  and  $\bar{I}(x)$  are the external sources for techniquark fields, respectively,  $\bar{\psi}_\xi(x)$  and  $\psi_\xi(x)$ ; and which leads to SDE for techniquark propagator,

$$S_X(x, y) = \langle X(x) \bar{X}(y) \rangle \quad X = U_\xi^l, D_\xi^l, U_\xi^t, D_\xi^t, U_\xi^b, D_\xi^b. \quad (33)$$

The detail derivation procedure is similar as that in Ref.[4]. The only difference is that now we have EF-FIETC in the theory. The final obtained SDE is

$$i\Sigma_X(x, y) = C_2(N) g_{\text{TC}}^2 G_{\mu\nu}(x, y) [\gamma^\mu S_X(x, y) \gamma^\nu] - iC_X \gamma_\mu [P_L S_X(x, x) P_L + P_R S_X(x, x) P_R] \gamma^\mu \delta(x - y), \quad (34)$$

with techniquark self energy defined as

$$i\Sigma_X(x, y) \equiv S_X^{-1}(x, y) + i[i\bar{\not{D}}_x + \bar{V}_\xi(x) + \bar{A}_\xi(x)\gamma_5] \delta(x - y), \quad (35)$$

and technigluon propagator  $G_{\mu\nu}^{\alpha\beta}(x, y) = \delta^{\alpha\beta} G_{\mu\nu}(x, y)$ .  $C_2(N) = (N^2 - 1)/(2N)$  is Casimir operator from  $(t^\alpha t^\alpha)_{ab} = C_2(N) \delta_{ab}$  for the fundamental representation of TC group  $SU(N)$ . Further  $C_X$  is effective ETC induced four fermion coupling which is

$$C_{U_\xi^l} = C_{U_\xi^t} = C_{U_\xi^b} = \frac{g_{\text{ETC}}^2}{M_{\text{ETC}}^2} b_U \quad (36)$$

$$C_{D_\xi^l} = C_{D_\xi^t} = C_{D_\xi^b} = \frac{g_{\text{ETC}}^2}{M_{\text{ETC}}^2} b_D.$$

In the following, we first consider the case of  $V_{\xi, \mu} = A_{\xi, \mu} = 0$ . In this situation, the technigluon propagator in Landau gauge is  $G_{\mu\nu}^{\alpha\beta}(x, y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} G_{\mu\nu}(p^2)$  with  $G_{\mu\nu}(p^2) = \frac{i}{-p^2[1+\Pi(-p^2)]} (g_{\mu\nu} - p_\mu p_\nu / p^2)$ . And the techniquark self energy and propagator are respectively

$$\begin{pmatrix} \Sigma_X(x, y) \\ S_X(x, y) \end{pmatrix} = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \times \begin{pmatrix} \Sigma_X(-p^2) \\ S_X(-p^2) \end{pmatrix}, \quad (37)$$

with  $S_X(p) = i/[p - \Sigma_X(-p^2)]$ . Substitute above results into the SDE and parameterize the technigluon propagator as  $\alpha_{\text{TC}}[(p_E - q_E)^2] \equiv g_{\text{TC}}^2/(4\pi[1 + \Pi(p_E^2)])$  for Euclidean momentum  $p_E, q_E$ , we obtain following integration equation which with angular approximation  $\alpha_{\text{TC}}[(p_E - q_E)^2] = \alpha_{\text{TC}}(p_E^2) \theta(p_E^2 - q_E^2) + \alpha_{\text{TC}}(q_E^2) \theta(q_E^2 - p_E^2)$ , can be further reduced to differential equation,

$$i\Sigma_X(-p^2) = 4 \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{3\pi C_2(N) \alpha_{\text{TC}}[-(p-q)^2]}{(p-q)^2} + C_X \right\} \times \left[ \frac{\Sigma_X(-q^2)}{q^2 - \Sigma_X^2(-q^2)} \right]. \quad (38)$$

Once above equation presents nonzero solution, we obtain nontrivial techniquark condensate

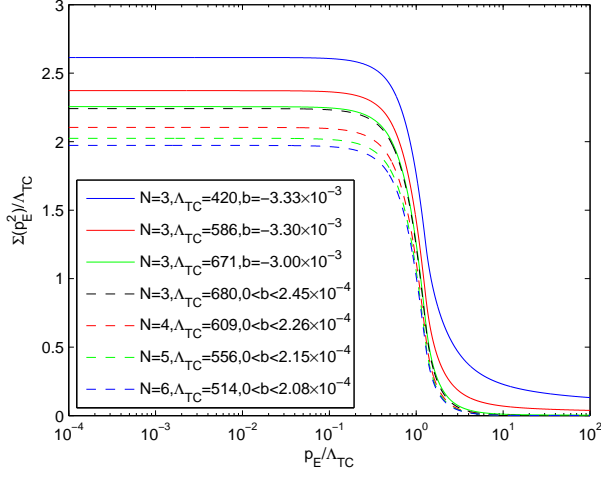
$$\langle \bar{X}(x) X'(x) \rangle = -4N \delta_{XX'} \int \frac{d^4p_E}{(2\pi)^4} \frac{\Sigma_X(p_E^2)}{p_E^2 + \Sigma_X^2(p_E^2)}, \quad (39)$$

which breaks  $SU(2)_L \otimes U(1)_1 \otimes U(1)_2$  to subgroup  $U(1)_{\text{em}}$ .

To obtain the numerical solution of equation (38), we take the running constant  $\alpha_{\text{TC}}(p^2)$  the same as that used in Eq.(49) of Ref.[4] for which there are four input parameters:  $N$ ,  $N_f$ ,  $\Lambda_{\text{TC}}$  and  $b$ .  $N$  as TC number is a free



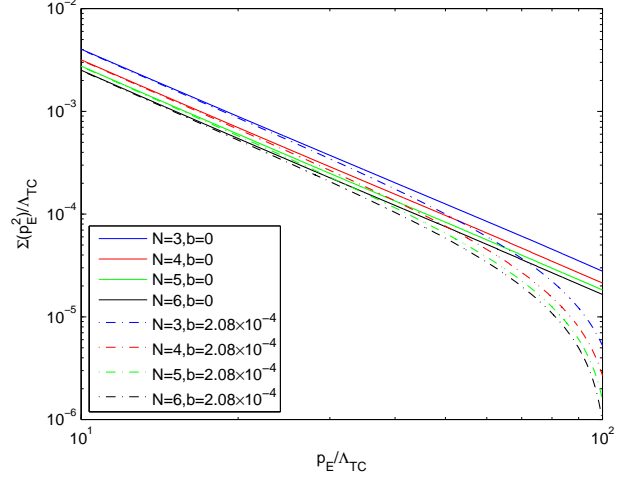
FIG. 1: Techniquark self energy  $\Sigma(p_E^2)$ .  $\Lambda_{\text{TC}}$  is in unit of GeV and is fixed by  $f = 250\text{GeV}$ .



parameter, we take four different values  $N = 3, 4, 5, 6$  estimating its effects;  $N_f = 6$  is due to three doublets of techniquarks; The scale of TC interaction  $\Lambda_{\text{TC}}$  will be fixed from  $f = 250\text{GeV}$  determined in later in (70);  $b \equiv C\Lambda_{\text{TC}}^2$  will be discussed later with  $C$  introduced in (36) as coefficients of EFFIETC. We take the physical cutoff of the equation to be the scale of ETC and  $\Lambda = \Lambda_{\text{ETC}} = 100\Lambda_{\text{TC}}$ . The result  $\Sigma(p_E^2)$  is depicted in Fig.1 in which dashed lines are for positive  $b$  and different  $N$ s; while solid lines are for different negative  $b$ s and  $N = 3$ . From which, we find

1. For  $N=3$  and positive  $b$ , EFFIETC infects  $\Sigma(p_E^2)$  very little except to its large momentum tail. We have changed coupling  $b$  by enlarging its magnitude 100 times, the general form of  $\Sigma(p_E^2)$  almost do not change. For  $N = 3$  and negative  $b$ , above  $b = -0.00300$  the change in  $\Sigma(p_E^2)$  is small, below  $b = -0.00300$ , we see the explicit change of  $\Sigma(p_E^2)$  which at large momentum region exhibits typical slowly damping asymptotic behavior due to existence of four fermion coupling. To check the validity of the phenomena, we have changed differential equation to original integration equation for SDE with and without angular approximation  $\alpha_{\text{TC}}[(p_E - q_E)^2] = \alpha_{\text{TC}}(p_E^2)\theta(p_E^2 - q_E^2) + \alpha_{\text{TC}}(q_E^2)\theta(q_E^2 - p_E^2)$  and increased the cutoff of the theory, all obtain the similar result. For  $N = 4, 5, 6$ , we can find similar phenomena as the case of  $N = 3$  which are not written down here, since later we will show that present model prefers smaller  $N$  and then the final result of our calculation will be only limited in the case of  $N = 3$ .
2. For large momentum tail of  $\Sigma(p_E^2)$ , we find that if positive  $b$  is larger than some critical value,  $\Sigma(p_E^2)$  will be negative as momentum becoming

FIG. 2: The tail of techniquark self energy  $\Sigma(p_E^2)$  exhibits ETC effects.



large which indicates the possible oscillation. These values are  $b_{N=3} = 2.45 \times 10^{-4}$ ,  $b_{N=4} = 2.26 \times 10^{-4}$ ,  $b_{N=5} = 2.15 \times 10^{-4}$ ,  $b_{N=6} = 2.08 \times 10^{-4}$ . Considering that  $b \propto \Lambda_{\text{TC}}^2/\Lambda_{\text{ETC}}^2$  must be very small, we take  $b = 2.08 \times 10^{-4}$  as a typical value of our computation. To exhibit the difference of tail for different  $b$ , we draw diagrams of  $\Sigma(p_E^2)$  with  $b = 2.08 \times 10^{-4}$  and  $b = 0$  together in Fig.2. We find that the differences show up only in the tail of self energy at momentum beyond  $50\Lambda_{\text{TC}}$  and below that limit, there is almost no difference. We further find that for fixed  $f = 250\text{GeV}$ , from later result of (70), both  $b = 0$  and  $b = 2.08 \times 10^{-4}$  cases all lead almost the same  $\Lambda_{\text{TC}}$ .

If we further take  $b_U = b_D$ ,  $\Sigma_X$  equals for each techniflavor and we can neglect subscript  $X$ . Then with technique developed in Ref.[4], we can show that if the function  $\Sigma(\partial_x^2)\delta(x-y)$  is the solution of the SDE in the case  $V_{\xi,\mu} = A_{\xi,\mu} = 0$ , we can replace its argument  $\partial_x$  by the minimal-coupling covariant derivative  $\bar{\nabla}_x \equiv \partial_x - iV_\xi(x)$  and use it, *i.e.*,  $\Sigma(\bar{\nabla}_x^2)\delta(x-y)$ , as an approximate solution of the SDE in the case  $V_{\xi,\mu} \neq 0$  and  $A_{\xi,\mu} \neq 0$ .

## 2. Effective Action

Starting from (32), the exponential multi-fermion terms on the right-hand side of equation can be written explicitly as

$$\sum_{n=2}^{\infty} \int d^4x_1 \dots d^4x_n \frac{(-ig_{\text{TC}})^n}{n!} G_{\mu_1 \dots \mu_n}^{\alpha_1 \dots \alpha_n}(x_1, \dots, x_n) J_{\xi, \alpha_1}^{\mu_1}(x_1) \dots J_{\xi, \alpha_n}^{\mu_n}(x_n) \approx \int d^4x d^4x' \bar{\psi}_{\xi}^{\sigma}(x) \Pi_{\sigma\rho}(x, x') \psi_{\xi}^{\rho}(x'), \quad (40)$$

$$\Pi_{\sigma\rho}(x, x') = \sum_{n=2}^{\infty} \Pi_{\sigma\rho}^{(n)}(x, x') \approx \Pi_{\sigma\rho}^2(x, x'), \quad (41)$$

$$\Pi_{\sigma\rho}^{(2)}(x, x') = -g_{\text{TC}}^2 G_{\mu_1\mu_2}^{\alpha_1\alpha_2}(x, x') \left[ t_{\alpha_1}\gamma^{\mu_1} S(x, x') t_{\alpha_2}\gamma^{\mu_2} \right]_{\sigma\rho} \quad (42)$$

where we have taken the approximation of *replacing the summation over 2n-fermion interactions with parts of them by their vacuum expectation values (VEVs) and only keeping the leading four fermion interactions*. For  $\mathcal{L}_{\xi 4T}$  term in (32), we use the same approximation given above. Combining with result (34) and neglecting the factor  $\mathcal{F}[O_\xi]\delta(O_\xi - O_\xi^\dagger)$  in Eq.(32), we obtain

$$\begin{aligned} S_{\text{norm}}[U, W_\mu^a, B_\mu] \\ \approx -i \log \int \mathcal{D}\bar{\psi}_\xi \mathcal{D}\psi_\xi \exp \left[ i \int d^4x \bar{\psi}_\xi (i\cancel{\partial} + \cancel{V}_\xi + \cancel{A}_\xi \gamma^5) \psi_\xi \right. \\ \left. - i \int d^4x d^4x' \bar{\psi}_\xi^\sigma(x) \Sigma_{\sigma\rho}(x, x') \psi_\xi^\rho(x') \right] \\ \approx -i \text{Tr} \log [i\cancel{\partial} + \cancel{V}_\xi + \cancel{A}_\xi \gamma^5 - \Sigma(\bar{\nabla}^2)], \end{aligned} \quad (43)$$

where  $\Sigma(\bar{\nabla}^2)$  in techniflavor space is block diagonal. Notice that the arguments of  $\text{Tr} \log$  are block diagonal which enable us to compute them block by blocks,

$$\begin{aligned} S_{\text{norm}}[U, W_\mu^a, B_\mu] \\ = \sum_{\eta=1}^3 -i \text{Tr} \log [i\cancel{\partial} + \cancel{\phi}^\eta + \cancel{\phi}^\eta \gamma^5 - \Sigma(\bar{\nabla}^{\eta,2})] \\ = \sum_{\eta=1}^3 \int d^4x \text{tr}_f \left[ (F_0^{1D})^2 a^{\eta 2} - \mathcal{K}_1^{1D} (d_\mu a_\mu^\eta)^2 \right. \\ \left. - \mathcal{K}_2^{1D} (d_\mu a_\mu^\eta - d_\nu a_\nu^\eta)^2 + \mathcal{K}_3^{1D} (a^{\eta 2})^2 + \mathcal{K}_4^{1D} (a_\mu^\eta a_\nu^\eta)^2 \right. \\ \left. - \mathcal{K}_{13}^{1D} V_{\mu\nu}^\eta V^{\eta\mu\nu} + i \mathcal{K}_{14}^{1D} a_\mu^\eta a_\nu^\eta V^{\eta\mu\nu} \right] + \mathcal{O}(p^6), \end{aligned} \quad (44)$$

for which  $\bar{\nabla}_\mu^\eta \equiv \partial_\mu - i v_\xi^\eta$  and from (15) to (18) and (25) to (27),

$$\begin{aligned} v_\mu^\eta &= -\frac{1}{2} g_2 \frac{\tau^a}{2} W_{\xi,\mu}^a - \frac{1}{2} g_1 \frac{\tau^3}{2} B_{\xi,\mu} + Z_{V\mu}^\eta, \\ a_\mu^\eta &= \frac{1}{2} g_2 \frac{\tau^a}{2} W_{\xi,\mu}^a - \frac{1}{2} g_1 \frac{\tau^3}{2} B_{\xi,\mu} + Z_{A\mu}^\eta \quad \eta = l, t, b, \end{aligned} \quad (45)$$

where  $d_\eta a_\nu^\eta \equiv \partial_\mu a_\nu^\eta - i[v_\mu^\eta, a_\nu^\eta]$ ,  $V_{\mu\nu}^\eta \equiv i[\partial_\mu - i v_\mu^\eta, \partial_\nu - i v_\nu^\eta]$ .  $F_0^{1D}$  and  $\mathcal{K}_i^{1D}$  coefficients with superscript 1D to denote that they are from one doublet TC model discussed in Ref.[4] which are functions of techniquark self energy  $\Sigma(p^2)$  and detailed expressions of them are already given in (36) of Ref.[9] with the replacement of  $N_c \rightarrow N$ .

For anomaly part,  $U$  field dependent part can be produced by normal part with vanishing techniquark self energy  $\Sigma$ , i.e.

$$\begin{aligned} i S_{\text{anom}}[U, W_\mu^a, B_\mu] \\ = \text{Tr} \log (i\cancel{\partial} + \cancel{V} + \cancel{A} \gamma^5) - i S_{\text{norm}}[U, W_\mu^a, B_\mu]|_{\Sigma=0}. \end{aligned} \quad (46)$$

Notice that pure gauge field part independent of  $U$  field is irrelevant to EWCL. Combined with (44), above relation imply

$$\begin{aligned} i S_{\text{anom}}[U, W_\mu^a, B_\mu] \\ = \text{Tr} \log (i\cancel{\partial} + \cancel{V} + \cancel{A} \gamma^5) + i \sum_{\eta=1}^3 \int d^4x \text{tr}_f \left[ -\mathcal{K}_1^{1D,(\text{anom})} (d_\mu a_\mu^\eta)^2 \right. \\ \left. - \mathcal{K}_2^{1D,(\text{anom})} (d_\mu a_\mu^\eta - d_\nu a_\nu^\eta)^2 + \mathcal{K}_3^{1D,(\text{anom})} (a^{\eta 2})^2 \right. \\ \left. + \mathcal{K}_4^{1D,(\text{anom})} (a_\mu^\eta a_\nu^\eta)^2 - \mathcal{K}_{13}^{1D,(\text{anom})} V_{\mu\nu}^\eta V^{\eta\mu\nu} \right. \\ \left. + i \mathcal{K}_{14}^{1D,(\text{anom})} a_\mu^\eta a_\nu^\eta V^{\eta\mu\nu} \right] + \mathcal{O}(p^6), \end{aligned} \quad (47)$$

with

$$\mathcal{K}_i^{1D,(\text{anom})} = -\mathcal{K}_i^{1D}|_{\Sigma=0} \quad i = 1, 2, 3, 4, 13, 14 \quad (48)$$

where we have used result that  $F_0^{1D}|_{\Sigma=0} = 0$ . Combining normal and anomaly part contributions together, with help of (29), we finally find

$$\begin{aligned} S_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu] \\ = \int d^4x \left( -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} \right. \\ \left. + \frac{1}{2} M_0^2 Z'_\mu Z'^\mu \right) - i \text{Tr} \log (i\cancel{\partial} + \cancel{V} + \cancel{A} \gamma^5) + \sum_{\eta=1}^3 \int d^4x \text{tr}_f \left[ \right. \\ (F_0^{1D})^2 a^{\eta 2} - \mathcal{K}_1^{1D,\Sigma \neq 0} (d_\mu a_\mu^\eta)^2 - \mathcal{K}_2^{1D,\Sigma \neq 0} (d_\mu a_\mu^\eta - d_\nu a_\nu^\eta)^2 \\ \left. + \mathcal{K}_3^{1D,\Sigma \neq 0} (a^{\eta 2})^2 + \mathcal{K}_4^{1D,\Sigma \neq 0} (a_\mu^\eta a_\nu^\eta)^2 - \mathcal{K}_{13}^{1D,\Sigma \neq 0} V_{\mu\nu}^\eta V^{\eta\mu\nu} \right. \\ \left. + i \mathcal{K}_{14}^{1D,\Sigma \neq 0} a_\mu^\eta a_\nu^\eta V^{\eta\mu\nu} \right] + \mathcal{O}(p^6). \end{aligned} \quad (49)$$

With help of (45) and (25) to (27), above result can be further simplified to the form (A4) in which explicitly  $U$  field dependence is displayed.

### C. Integrating out $Z'$

We can further decompose (A4) into

$$\begin{aligned} S_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu] \\ = \tilde{S}_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu] + S_{Z'}[U, W_\mu^a, B_\mu, 0], \end{aligned} \quad (50)$$

where  $\tilde{S}_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu]$  is  $Z'$  dependent part of  $S_{\text{eff}}[U, W_\mu^a, B_\mu, Z'_\mu]$ . We find  $Z'$  independent part  $S_{Z'}[U, W_\mu^a, B_\mu, 0]$  is just the same as that of one-doublet TC model given in Ref.[4], the only difference is that now there is an extra overall factor 3 multiplied in front of all terms. The source of this factor comes from the fact that in present model, instead of one doublet, we have three techniquark doublets. So Switching off effects from  $Z'$  particle, contributions of present TC2 model to bosonic part of EWCL are equivalent to those of three-doublets

TC model. In  $\tilde{S}_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu]$ , in order to normalize  $Z'$  field correctly, we introduce normalized field  $Z'_{R,\mu}$  as

$$\begin{aligned} Z'_\mu &= \frac{1}{c_{Z'}} Z'_{R,\mu} \\ c_{Z'}^2 &= 1 + g_1^2 [3\mathcal{K} \tan^2 \theta' + 10\mathcal{K}(\tan \theta' + \cot \theta')^2 \\ &\quad + \mathcal{K}_2^{1D,\Sigma \neq 0}(\tan \theta' + \cot \theta')^2 + \frac{3}{2}\mathcal{K}_2^{1D,\Sigma \neq 0} \tan^2 \theta' \\ &\quad + \frac{9}{2}\mathcal{K}_{13}^{1D,\Sigma \neq 0}(\tan \theta' + \cot \theta')^2 + \frac{3}{2}\mathcal{K}_{13}^{1D,\Sigma \neq 0} \tan^2 \theta'] , \end{aligned} \quad (51)$$

in terms of normalized field  $Z'_{R,\mu}$ ,  $\tilde{S}_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu]$  become

$$\begin{aligned} \tilde{S}_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu] &= \int d^4x \left[ \frac{1}{2} Z'_{R,\mu} D_Z^{-1,\mu\nu} Z'_{R,\nu} \right. \\ &\quad \left. + Z'_{R,\mu} J_{Z,\mu} + Z_R^2 Z'_{R,\mu} J_{3Z}^\mu + g_{4Z} \frac{g_1^4}{c_{Z'}^4} Z_R'^{4,4} \right] \end{aligned} \quad (52)$$

with

$$D_Z^{-1,\mu\nu} = g^{\mu\nu}(\partial^2 + M_{Z'}^2) - (1 + \lambda_Z)\partial^\mu \partial^\nu + \Delta_Z^{\mu\nu}(X) , \quad (53)$$

$$\begin{aligned} M_{Z'}^2 &= \frac{1}{c_{Z'}^2} \{ M_0^2 + \frac{1}{2}(F_0^{1D})^2 g_1^2 (\cot \theta' + \tan \theta')^2 \\ &\quad + \frac{3}{4}(F_0^{1D})^2 g_1^2 \tan^2 \theta' \} , \end{aligned} \quad (54)$$

$$\lambda_Z = \frac{g_1^2}{c_{Z'}^2} \left[ -\frac{1}{2}(\tan \theta' + \cot \theta')^2 - \frac{3}{4} \tan^2 \theta' \right] \mathcal{K}_1^{1D,\Sigma \neq 0} , \quad (55)$$

$$\begin{aligned} \Delta_Z^{\mu\nu}(X) &= \frac{1}{c_{Z'}^2} \left\{ \left( -\frac{3}{4}\mathcal{K}_1^{1D,\Sigma \neq 0} - \frac{3}{16}\mathcal{K}_3^{1D,\Sigma \neq 0} + \frac{3}{8}\mathcal{K}_{13}^{1D,\Sigma \neq 0} \right. \right. \\ &\quad \left. - \frac{3}{16}\mathcal{K}_{14}^{1D,\Sigma \neq 0} \right) g_1^2 \tan^2 \theta' \text{tr}[X^\mu \tau^3] \text{tr}[X^\nu \tau^3] \\ &\quad + \left[ \frac{3}{2}\mathcal{K}_1^{1D,\Sigma \neq 0} \tan^2 \theta' - \frac{1}{4}(\cot \theta' + \tan \theta')^2 \mathcal{K}_3^{1D,\Sigma \neq 0} \right. \\ &\quad \left. - \frac{1}{4}(\cot \theta' + \tan \theta')^2 \mathcal{K}_4^{1D,\Sigma \neq 0} - \frac{3}{8}\mathcal{K}_4^{1D,\Sigma \neq 0} \tan^2 \theta' \right. \\ &\quad \left. - \frac{3}{4}\mathcal{K}_{13}^{1D,\Sigma \neq 0} \tan^2 \theta' + \frac{3}{8}\mathcal{K}_{14}^{1D,\Sigma \neq 0} \tan^2 \theta' \right] g_1^2 \text{tr}[X^\mu X^\nu] \\ &\quad + g^{\mu\nu} \left[ \left( -\frac{1}{8}(\cot \theta' + \tan \theta')^2 - \frac{3}{16} \tan^2 \theta' \right) \mathcal{K}_3^{1D,\Sigma \neq 0} \right. \\ &\quad \left. + \frac{3}{16} \tan^2 \theta' \mathcal{K}_4^{1D,\Sigma \neq 0} - \frac{1}{8}(\cot \theta' + \tan \theta')^2 \mathcal{K}_4^{1D,\Sigma \neq 0} \right. \\ &\quad \left. + \frac{3}{4} \tan^2 \theta' \mathcal{K}_{13}^{1D,\Sigma \neq 0} - \frac{3}{8} \tan^2 \theta' \mathcal{K}_{14}^{1D,\Sigma \neq 0} \right] g_1^2 \text{tr}[X^k X_k] \\ &\quad \left. + g^{\mu\nu} \left[ -\frac{3}{16}\mathcal{K}_4^{1D,\Sigma \neq 0} - \frac{3}{8}\mathcal{K}_{13}^{1D,\Sigma \neq 0} + \frac{3}{16}\mathcal{K}_{14}^{1D,\Sigma \neq 0} \right] \right\} \\ &\quad \times g_1^2 \tan^2 \theta' \text{tr}[X_k \tau^3] \text{tr}[X^k \tau^3] , \end{aligned} \quad (56)$$

$$J_Z^\mu = J_{Z0}^\mu + \frac{g_1^2 \gamma}{c_{Z'}} \partial^\nu B_{\mu\nu} + \tilde{J}_Z^\mu , \quad (57)$$

$$J_{Z0\mu} = -\frac{3}{4c_{Z'}} i(F_0^{1D})^2 g_1 \tan \theta' \text{tr}[X_\mu \tau^3] , \quad (58)$$

$$\gamma = 3\mathcal{K} \tan \theta' + \left( \frac{3}{2}\mathcal{K}_2^{1D,\Sigma \neq 0} + \frac{3}{2}\mathcal{K}_{13}^{1D,\Sigma \neq 0} \right) \tan \theta' , \quad (59)$$

$$\begin{aligned} \tilde{J}_Z^\mu &= \frac{1}{c_{Z'}} \left\{ \frac{3}{4} i g_1 \tan \theta' \mathcal{K}_1^{1D,\Sigma \neq 0} \{ \text{tr}[U^\dagger (D^\nu D_\nu U) U^\dagger D^\mu U \tau^3] \right. \\ &\quad \left. - \tan \theta' \text{tr}[U^\dagger (D^\nu D_\nu U) \tau^3 U^\dagger D^\mu U + \partial^\mu (U^\dagger D^\nu D_\nu U \tau^3)] \right\} \\ &\quad + \frac{3}{2} (-\mathcal{K}_2^{1D,\Sigma \neq 0} + \mathcal{K}_{13}^{1D,\Sigma \neq 0}) g_1 \tan \theta' \partial_\nu \text{tr}[\bar{W}^{\mu\nu} \tau^3] \\ &\quad + \frac{3i}{4} \left( \frac{1}{4}\mathcal{K}_3^{1D,\Sigma \neq 0} - \frac{1}{4}\mathcal{K}_4^{1D,\Sigma \neq 0} - \mathcal{K}_{13}^{1D,\Sigma \neq 0} + \frac{1}{2}\mathcal{K}_{14}^{1D,\Sigma \neq 0} \right) \\ &\quad \times g_1 \tan \theta' \text{tr}[X^\nu X_\nu] \text{tr}[X^\mu \tau^3] + \frac{3i}{4} \left( \frac{1}{2}\mathcal{K}_4^{1D,\Sigma \neq 0} \right. \\ &\quad \left. + \mathcal{K}_{13}^{1D,\Sigma \neq 0} - \frac{1}{2}\mathcal{K}_{14}^{1D,\Sigma \neq 0} \right) g_1 \tan \theta' \text{tr}[X^\mu X_\nu] \text{tr}[X^\nu \tau^3] \\ &\quad + \frac{3}{4} (-\mathcal{K}_{13}^{1D,\Sigma \neq 0} + \frac{1}{4}\mathcal{K}_{14}^{1D,\Sigma \neq 0}) g_1 \tan \theta' \text{tr}[\bar{W}^{\mu\nu} (X_\nu \tau^3 - \tau^3 X_\nu)] \\ &\quad \left. + \frac{3}{2} i (\mathcal{K}_{13}^{1D,\Sigma \neq 0} - \frac{1}{4}\mathcal{K}_{14}^{1D,\Sigma \neq 0}) g_1 \tan \theta' \partial_\nu \text{tr}[X^\mu X^\nu \tau^3] \right\} , \end{aligned} \quad (60)$$

$$\begin{aligned} g_{4Z} &= (\mathcal{K}_3^{1D,\Sigma \neq 0} + \mathcal{K}_4^{1D,\Sigma \neq 0}) \left[ \frac{3}{128} \tan^4 \theta' \right. \\ &\quad \left. + \frac{3}{32} \tan^2 \theta' (\cot \theta' + \tan \theta')^2 + \frac{1}{64} (\cot \theta' + \tan \theta')^4 \right] , \end{aligned} \quad (61)$$

$$\begin{aligned} J_{3Z}^\mu &= \frac{-i}{c_{Z'}^3} (\mathcal{K}_3^{1D,\Sigma \neq 0} + \mathcal{K}_4^{1D,\Sigma \neq 0}) g_1^3 \left[ \frac{3}{32} \tan^3 \theta' \right. \\ &\quad \left. + \frac{3}{16} (\cot \theta' + \tan \theta')^2 \tan \theta' \right] \text{tr}[X^\mu \tau_3] . \end{aligned} \quad (62)$$

Perform loop expansion to (22), the result of  $Z'$  field integration is

$$\begin{aligned} S_{\text{eff}}[U, W_\mu^a, B_\mu] &- i \log \mathcal{N}[W_\mu^a, B_\mu] \\ &= \tilde{S}_{Z'}[Z'_c, U, W^a, B] + \text{loop terms} , \end{aligned} \quad (63)$$

with classical field  $Z'_c$  satisfy

$$\frac{\partial}{\partial Z'_{c,\mu}(x)} \left[ \tilde{S}_{Z'}[Z'_c, U, W^a, B] + \text{loop terms} \right] = 0 , \quad (64)$$

and

$$-i \log \mathcal{N}[W_\mu^a, B_\mu] = \left[ \tilde{S}_{Z'}[Z'_c, U, W^a, B] + \text{loop terms} \right]_{\Sigma=0} \quad (65)$$

which is obtained from (13) and the fact that when we switch off TC and ETC interactions, techniquark self energy vanishes. With (52), the solution is

$$Z'_c{}^\mu(x) = -D_Z^{\mu\nu} J_{Z,\nu}(x) + O(p^3) + \text{loop terms} . \quad (66)$$

Then

$$\begin{aligned} S_{\text{eff}}[U, W_\mu^a, B_\mu] &- i \log \mathcal{N}[W_\mu^a, B_\mu] \\ &= \int d^4x \left[ -\frac{1}{2} J_{Z,\mu} D_Z^{\mu\nu} J_{Z,\nu} - J_{3Z,\mu'} (D_Z^{\mu'\nu'} J_{Z,\nu'}) (D_Z^{\mu\nu} J_{Z,\nu})^2 \right. \\ &\quad \left. + g_{4Z} \frac{g_1^4}{c_{Z'}} (D_Z^{\mu\nu} J_{Z,\nu})^4 \right] + \text{loop terms} , \end{aligned} \quad (67)$$

where  $D_Z^{-1,\mu\nu} D_{Z,\nu\lambda} = D_Z^{\mu\nu} D_{Z,\nu\lambda}^{-1} = g_\lambda^\mu$  and it is not difficult to show that if we are accurate up to order of  $p^4$ , then order  $p$  of  $Z'_c$  solution is enough, all contributions



from order  $p^3$  of  $Z'_c$  are at least belong to order of  $p^6$ . With help of (67), (53) and (57)

$$S_{\text{eff}}[U, W_\mu^a, B_\mu] - i \log \mathcal{N}[W_\mu^a, B_\mu] \\ = \int d^4x \left[ -\frac{1}{2} J_{Z0,\mu} D_Z^{\mu\nu} J_{Z0,\nu} - \frac{1}{M_{Z'}^2} J_{Z0,\mu} (\tilde{J}_Z^\mu + \frac{g_1^2 \gamma}{c_{Z'}} \partial_\nu B^{\mu\nu}) \right. \\ \left. - \frac{1}{M_{Z'}^6} J_{3Z,\mu} J_{Z0}^\mu J_{Z0}^2 + \frac{g_{4Z} g_1^4}{c_{Z'}^4 M_{Z'}^8} J_{Z0}^4 \right]. \quad (68)$$

Ignoring terms higher than order of  $p^4$ , we find  $S_{\text{eff}}[U, W_\mu^a, B_\mu]$  has exact form of standard EWCL up to order of  $p^4$ . We can then read out the corresponding coefficients, the result will be given in next subsection. The normalization factor now is

$$-i \log \mathcal{N}[W_\mu^a, B_\mu] = \int d^4x \left[ -\left(\frac{1}{4} + \frac{3}{4} \mathcal{K} g_2^2 + \frac{3}{8} \mathcal{K}_2^{1D, \Sigma \neq 0} g_2^2 \right. \right. \\ \left. \left. + \frac{3}{8} \mathcal{K}_{13}^{1D, \Sigma \neq 0} g_2^2 \right) W_{\mu\nu}^a W^{a, \mu\nu} - \left(\frac{1}{4} + \frac{3}{4} \mathcal{K} g_1^2 \right. \right. \\ \left. \left. + \frac{3}{8} \mathcal{K}_2^{1D, \Sigma \neq 0} g_1^2 + \frac{3}{8} \mathcal{K}_{13}^{1D, \Sigma \neq 0} g_1^2 + \frac{3(F_0^{1D})^2}{8M_{Z'}^2} \beta_1 g_1^2 \right. \right. \\ \left. \left. + \beta_1 g_1^2 \cot \theta' \gamma \right) B_{\mu\nu} B^{\mu\nu} \right]. \quad (69)$$

#### D. Coefficients of EWCL

From  $S_{\text{eff}}[U, W_\mu^a, B_\mu]$  obtained in last subsection, we can read out coefficients of EWCL. The  $p^2$  order coefficients are

$$f^2 = 3(F_0^{1D})^2 \quad \beta_1 = \frac{3(F_0^{1D})^2 g_1^2 \tan^2 \theta'}{8c_{Z'}^2 M_{Z'}^2}. \quad (70)$$

Combining with (10), (54) and  $T$  parameter  $\alpha T = 2\beta_1$  given in Ref.[2], we further obtain

$$\beta_1 = \frac{1}{2} \alpha T = \frac{12}{(\frac{200v^2}{3f^2} + 16)(1 + \cot^2 \theta')^2 + 24}, \quad (71)$$

then  $T$  is positive and uniquely determined by  $\theta'$  and  $v/f$ . It is bounded above and the upper limit is  $3/(5 + 25v^2/3f^2)\alpha \leq 9/(40\alpha)$ , since we know  $v \geq f$ . In following numerical computations, for simplicity, we all take  $v = f$ .  $p^4$  order coefficients are

$$\alpha_1 = 3(1 - 2\beta_1)L_{10}^{1D} + \frac{3(F_0^{1D})^2}{2M_{Z'}^2} \beta_1 - 2\gamma\beta_1 \cot \theta', \\ \alpha_2 = -\frac{3}{2}(1 - 2\beta_1)L_9^{1D} + \frac{3(F_0^{1D})^2}{2M_{Z'}^2} \beta_1 - 2\gamma\beta_1 \cot \theta', \\ \alpha_3 = -\frac{3}{2}(1 - 2\beta_1)L_9^{1D}, \\ \alpha_4 = 3L_2^{1D} + 6\beta_1 L_9^{1D} + \frac{3(F_0^{1D})^2}{2M_{Z'}^2} \beta_1, \\ \alpha_5 = 3L_1^{1D} + \frac{3}{2}L_3^{1D} - \frac{3(F_0^{1D})^2}{2M_{Z'}^2} \beta_1 - 6\beta_1 L_9,$$

$$\alpha_6 = -\frac{3(F_0^{1D})^2}{2M_{Z'}^2} \beta_1 - 6\beta_1(4L_1^{1D} + L_9^{1D}) \\ + \beta_1^2[(1 + \cot^2 \theta')^2(48L_1^{1D} + 8L_3^{1D}) + 24L_1^{1D}], \\ \alpha_7 = \frac{3(F_0^{1D})^2}{2M_{Z'}^2} \beta_1 - 2\beta_1(3L_3^{1D} + 6L_1^{1D} - 3L_9^{1D}) + \beta_1^2[(1 \\ + \cot^2 \theta')^2(24L_1^{1D} + 4L_3^{1D}) + 6 \tan \theta'(L_3^{1D} + 2L_1^{1D})], \\ \alpha_8 = -\frac{3(F_0^{1D})^2}{2M_{Z'}^2} \beta_1 + 12\beta_1 L_{10}^{1D}, \quad (72) \\ \alpha_9 = -\frac{3(F_0^{1D})^2}{2M_{Z'}^2} \beta_1 + 6\beta_1(L_{10}^{1D} - L_9^{1D}), \\ \alpha_{10} = 4\beta_1^2(18L_1^{1D} + 3L_3^{1D}) + 32\beta_1^4 g_{4Z} \cot^4 \theta' \\ - \beta_1^3(144L_1^{1D} + 24L_3^{1D})[1 + 2(1 + \cot^2 \theta')^2], \\ \alpha_{11} = \alpha_{12} = \alpha_{13} = \alpha_{14} = 0,$$

where  $L_i$  relate to  $\mathcal{K}_i^{1D, \Sigma \neq 0}$  coefficients through

$$\mathcal{K}_2^{1D, \Sigma \neq 0} = L_{10}^{1D} - 2H_1^{1D}, \\ \mathcal{K}_3^{1D, \Sigma \neq 0} = 64L_1^{1D} + 16L_3^{1D} + 8L_9^{1D} + 2L_{10}^{1D} + 4H_1^{1D}, \\ \mathcal{K}_4^{1D, \Sigma \neq 0} = 32L_1^{1D} - 8L_9^{1D} - 2L_{10}^{1D} - 4H_1^{1D}, \\ \mathcal{K}_{13}^{1D, \Sigma \neq 0} = -L_{10}^{1D} - 2H_1^{1D}, \\ \mathcal{K}_{14}^{1D, \Sigma \neq 0} = -4L_{10}^{1D} - 8L_9^{1D} - 8H_1^{1D}. \quad (73)$$

Several features of this result are:

1. The contributions to  $p^4$  order coefficients are divided into two parts: a three doublets TC model contribution (equals to three times of one doublet TC model discussed in Ref.[4]) and  $Z'$  contribution.
2. All corrections from  $Z'$  particle are at least proportional to  $\beta_1$  which vanish if the mixing disappear by  $\theta' = 0$ .
3. Since  $L_{10}^{1D} < 0$ , combining with positive  $\beta_1$ , (72) then tells us  $\alpha_8$  is negative. Then  $U = -16\pi\alpha_8$  coefficient given in Ref.[2] is always positive in present model.
4.  $\alpha_1$  and  $\alpha_2$  depend on  $\gamma$  which from (59) further rely on an extra parameter  $\mathcal{K}$ . We can combine (70) and (51) together to fix  $\mathcal{K}$ ,

$$\frac{(F_0^{1D})^2 g_1^2 \tan^2 \theta'}{8\beta_1 M_{Z'}^2} \\ = \frac{1}{3} + g_1^2 \mathcal{K} \tan^2 \theta' + \frac{10}{3} \mathcal{K} (\tan \theta' + \cot \theta')^2 \\ + \frac{1}{3} \mathcal{K}_2^{1D, \Sigma \neq 0} (\tan \theta' + \cot \theta')^2 + \frac{1}{2} \mathcal{K}_2^{1D, \Sigma \neq 0} \tan^2 \theta' \\ + \frac{3}{2} \mathcal{K}_{13}^{1D, \Sigma \neq 0} (\tan \theta' + \cot \theta')^2 + \frac{1}{2} \mathcal{K}_{13}^{1D, \Sigma \neq 0} \tan^2 \theta'. \quad (74)$$

Once  $\mathcal{K}$  is fixed, with help of (A5), we can determine the ratio of infrared cutoff  $\kappa$  and ultraviolet cutoff  $\Lambda$ , in Fig.3, we draw the  $\kappa/\Lambda$  as function of  $T$  and  $M_{Z'}$ , we find natural criteria  $\Lambda > \kappa$

FIG. 3: The ratio of infrared cutoff and ultraviolet cutoff  $\kappa/\Lambda$  as function of  $T$  parameter and  $Z'$  mass in unit of TeV.

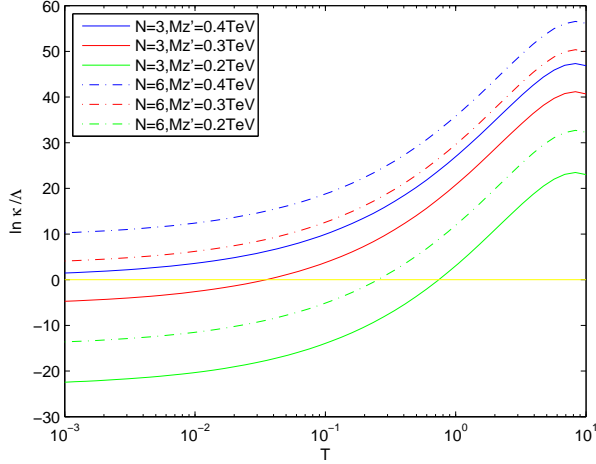
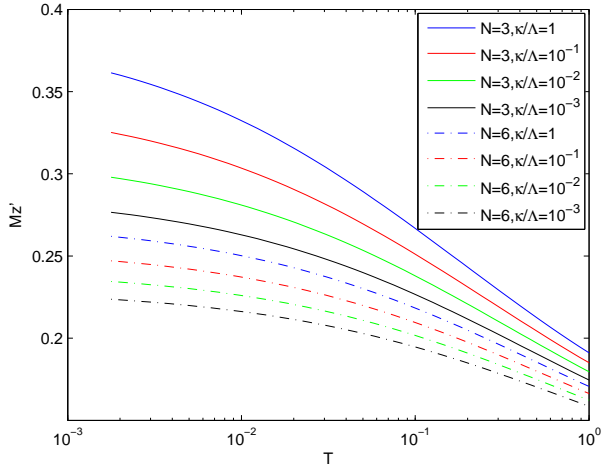
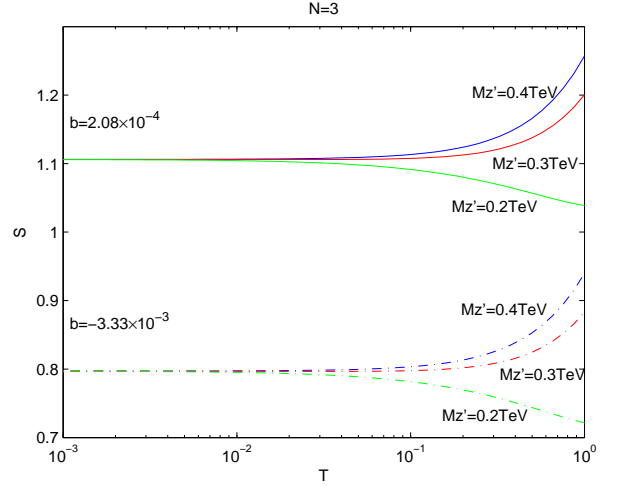


FIG. 4:  $Z'$  mass in unit of TeV as function of  $T$  parameter and  $\kappa/\Lambda$ .



offers stringent constraints on the allowed region for  $T$  and  $M_{Z'}$  that present theory prefer small  $Z'$  mass ( $< 0.4\text{TeV}$ ) and small TC group. For example,  $T < 0.035$  for  $M_{Z'} = 0.3\text{TeV}$  and  $N = 3$ ,  $T < 0.25$  for  $M_{Z'} = 0.2\text{TeV}$  and  $N = 6$ ,  $T < 0.74$  for  $M_{Z'} = 0.2\text{TeV}$  and  $N = 3$ . In Fig.4, we draw  $Z'$  mass as function of  $T$  parameter and  $\kappa/\Lambda$ . The line of  $\kappa/\Lambda = 1$  gives the upper bound of  $Z'$  mass  $M_{Z'} < 0.4\text{TeV}$ , which is already beyond the experiment limit given by Ref.[10, 11]. To check whether this bound is reliable, we have changed coupling of EFFHETC by either enlarging its magnitude 100 times or reversing its sign, the results all almost do not change. The special case of  $b = -3.33 \times 10^{-3}$  also has no effects here. To examine the reason

FIG. 5:  $S$  parameter for Lane's natural TC2 model.



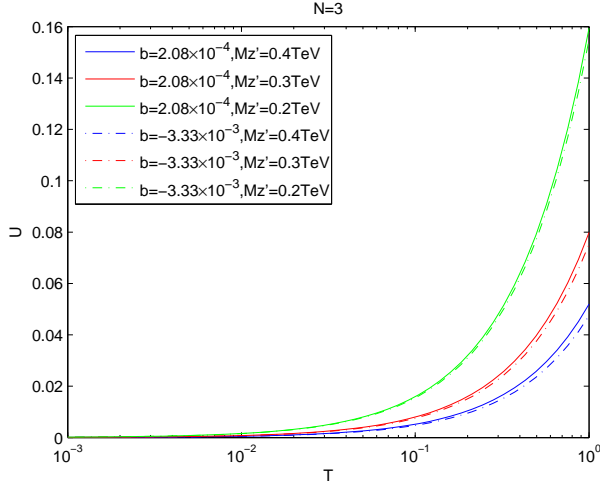
that why present model cause smaller  $M_{Z'}$  than that from Hill's model, we consider the situation of very tiny  $\theta'$  and  $\kappa/\Lambda$ , then the leading term in r.h.s. of (74) is  $\frac{10}{3}g_1^2\mathcal{K}\cot^2\theta'$ . Combining with Eq.(71), we find that (74) in this extreme case gives  $M_{Z'} = F_0\sqrt{\frac{31}{120}\mathcal{K}} \simeq f\sqrt{\mathcal{K}}/(2\sqrt{3})$ . while for Hill's model, we obtain result that  $M_{Z'} = F_0\sqrt{\mathcal{K}}/2 = f\sqrt{\mathcal{K}}/2$ . So  $Z'$  mass is smaller than that in Hill's model by a factor  $1/\sqrt{3}$  due to identification of  $F_0$  with  $f/\sqrt{3}$  now in (70) but with  $f$  in Hill's model. Considering smaller TC group will allow relative larger  $Z'$  mass, in following discussions, we only limit us in the case of  $N = 3$ .

5. For typical case with  $b = -3.33 \times 10^{-3}$ , except coefficients  $F_0^{1D}$  and  $\mathcal{K}_1^{1D}$  which receive relative large corrections from ETC interaction, all other  $\mathcal{K}_i^{1D}$  coefficients only feel small ETC effects.

With  $f = 250\text{GeV}$ , then all EWCL coefficients depend on two physical parameters  $\beta_1$  and  $M_{Z'}$ . Combined with  $\alpha T = 2\beta_1$ , we can use the present experimental result for the  $T$  parameter to fix  $\beta_1$ . In Fig.5 and 6, we draw graphs for the  $S = -16\pi\alpha_1$  and  $U = -16\pi\alpha_8$  in terms of the  $T$  parameter respectively. We take three typical  $Z'$  masses  $M_{Z'} = 0.2, 0.3, 0.4\text{TeV}$  for references.

For  $S$  parameter, we find that all values of it are at order of 1. This can be understood as that at region of small  $T$  parameter, the main contribution to  $S$  parameter comes from the three doublets TC model which results in positive  $S$ , roughly equals to  $-3L_{10}^{1D}$  which is three times larger than corresponding value in Hill's model due to existence of three doublets techniquarks. We also find that large negative  $b$  will reduce the value of  $S$ , but consider the value  $b = -0.00333$  corresponding to  $g_{\text{ETC}}^2 b_U = -0.00333\Lambda_{\text{ETC}}^2/\Lambda_{\text{TC}}^2$  is already large enough, we do not expect more negative larger  $b$  will have any physical meaning. For  $U$  parameter, we find it is posi-

FIG. 6:  $U$  parameter for Lane's natural TC2 model.



tive and below 0.2. Considering that the facts of small  $M_{Z'}$  and relative large  $S$  are all not favored by present precision measurements of SM, we just leave the analytic formulae for other  $\alpha_i$  coefficients there and will not draw diagrams for them further more.

### III. DISCUSSION

In this paper, we generalize the calculation in Ref.[4] for C.T.Hill's schematic TC2 model to K.Lane's prototype natural TC2 model. We find that, similar as Hill's model, coefficients of EWCL for the Lane's model are divided into direct TC and ETC interaction part, TC and topcolor induced effective  $Z'$  particle contribution part and ordinary quarks contribution part. The first two parts are computed in this paper. We show that the direct TC and ETC interaction part is three times larger than corresponding part of Hill's model due to existence of three techniquark doublets, while effective  $Z'$  contributions are different with Hill's model due to change of  $U(1)_1 \otimes U(1)_2$  group representation arrangements and are at least proportional to the  $p^2$  order parameter  $\beta_1$  in EWCL. Typical features of the model are that it only allows positive  $S$ ,  $T$  and  $U$  parameters.  $S$  is around 1 which is roughly three times larger than that in original Hill's model due to existence of three doublets of techniquarks, and  $T$  parameter varies in the range  $0 \sim 9/(40\alpha)$ . Analytical expression (72) for five  $p^4$  order coefficients including all three custodial symmetry conserve ones  $\alpha_3, \alpha_4, \alpha_5, \alpha_8, \alpha_9$  exactly equal to three times of those obtained from Hill's model in Ref.[4]. The  $Z'$  mass is bounded from 0.4TeV and larger  $M_{Z'}$  prefers smaller  $N$ . Compare to results obtained in Ref.[4] for C.T.Hill's TC2 model, the results from Lane's first natural TC2 model deviate more from the experiment data. This calls up for improvement of the model.

In fact, present model is only a prototype natural TC2

model. Many details of the model are even not specified in original paper [6] which prohibit us to perform computation more accurately and leave us more space to improve the dynamics. One typical non-specified effect is the walking dynamics. As mentioned by K.Lane that the TC of the model is expected to be a walking gauge theory. This is of new feature different with conventional gauge theory, and this walking is not explicitly realized in present prototype model, since techniquarks are in fundamental representation of TC group and number of techniquarks is not large enough. Another unspecified detail is the  $SU(3)_1 \otimes SU(3)_2$  symmetry breaking mechanism. It now is simulated without detail dynamics content by introducing an effective scalar field  $\Phi$  which transforms as  $(\bar{3}, 3, \frac{5}{6}, -\frac{5}{6})$  under the group  $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2}$  and corresponding interaction potential  $V(\Phi)$ . Introducing scalar fields, although now only is effective, deviates the basic idea of TC models. All these shortcomings are overcome in late improved model [8]. Considering that this new model is much more complex and different than present one which involves more of different dynamics and then requires more analysis and computation techniques. For example, the condensates of the techniquarks are block diagonal in three doublets flavor space now but not in the improved model (which is more like the case A solution of the paper [6], while present paper we only discuss the case B solution and ignore case A as mentioned in footnote <sup>2</sup>). In order to make our discussion not too much complex and specially exhibit the result for Lane's first natural TC2 model, in this paper, we only limit ourself in the primary prototype model and focus our attention on figuring out analytical expression for coefficients of EWCL, estimating possible constraints to the model and identifying the effects of ETC interactions, we leave the discussion of that new improved model in future paper.

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### APPENDIX A: NECESSARY FORMULAE FOR EWCL

In this appendix, we list down the necessary formulae needed in the text. With definition in which

$$D_\mu U = \partial_\mu U + ig_2 \frac{\tau^a}{2} W_\mu^a U - ig_1 U \frac{\tau^3}{2} B_\mu, \quad (A1)$$

$$D_\mu U^\dagger = \partial_\mu U^\dagger - ig_2 U^\dagger \frac{\tau^a}{2} W_\mu^a + ig_1 \frac{\tau^3}{2} B_\mu U^\dagger, \quad (A2)$$

$$X_\mu = U^\dagger (D_\mu U) \quad \overline{W}_{\mu\nu} = U^\dagger g_2 \frac{\tau^a}{2} W_{\mu\nu}^a U. \quad (A3)$$

we have

$$\begin{aligned}
& S_{Z'}[U, W_\mu^a, B_\mu, Z'_\mu] \\
&= \int d^4x \left\{ -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + \frac{1}{2} M_0^2 Z'_\mu Z'^\mu - \mathcal{K} \left[ \frac{3}{4} g_1^2 B_{\mu\nu} B^{\mu\nu} - \frac{3}{2} g_1^2 \tan \theta' B_{\mu\nu} Z'^{\mu\nu} \right. \right. \\
&\quad + \frac{3}{4} g_1^2 \tan^2 \theta' Z'_{\mu\nu} Z'^{\mu\nu} + \frac{5}{2} g_1^2 (\tan \theta' + \cot \theta')^2 Z'_{\mu\nu} Z'^{\mu\nu} + \frac{3}{4} g_2^2 W_{\mu\nu}^a W^{a,\mu\nu} \left. \right] + (F_0^{1D})^2 \left\{ -\frac{3}{4} \text{tr}[X^\mu X_\mu] \right. \\
&\quad + \frac{1}{4} g_1^2 (\cot \theta' + \tan \theta')^2 Z'^2 + \frac{3}{8} g_1^2 \tan^2 \theta' Z'^2 - i \frac{3}{4} g_1 \tan \theta' Z'^\mu \text{tr}[X_\mu \tau^3] \left. \right\} - \mathcal{K}_1^{1D, \Sigma \neq 0} \left\{ -\frac{3}{4} \text{tr}[U^\dagger (D^\mu D_\mu U) U^\dagger (D^\nu D_\nu U) \right. \\
&\quad + 2 U^\dagger (D^\mu D_\mu U) (D^\nu U^\dagger) (D_\nu U) \left. \right] - \frac{3}{4} i g_1 \tan \theta' Z'^\nu \text{tr}[U^\dagger (D^\mu D_\mu U) U^\dagger D_\nu U \tau^3] + \frac{3}{4} i g_1 \tan \theta' Z'^\nu \text{tr}[U^\dagger (D^\mu D_\mu U) \tau^3 U^\dagger D_\nu U] \\
&\quad - \frac{3}{4} i g_1 \tan \theta' \partial_\nu Z'^\nu \text{tr}[U^\dagger (D^\mu D_\mu U) \tau^3] \left. \right\} + \frac{3}{8} (\mathcal{K}_1^{1D, \Sigma \neq 0} + \frac{1}{4} \mathcal{K}_3^{1D, \Sigma \neq 0} - \frac{1}{4} \mathcal{K}_4^{1D, \Sigma \neq 0} - \mathcal{K}_{13}^{1D, \Sigma \neq 0} + \frac{1}{2} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) \\
&\quad \times [\text{tr}(X^\mu X_\mu)]^2 - \frac{3}{8} (\mathcal{K}_1^{1D, \Sigma \neq 0} + \frac{1}{4} \mathcal{K}_3^{1D, \Sigma \neq 0} - \frac{1}{2} \mathcal{K}_{13}^{1D, \Sigma \neq 0} + \frac{1}{4} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) g_1^2 \tan^2 \theta' Z'^\mu Z'_\nu \text{tr}[X_\mu \tau^3] \text{tr}[X^\nu \tau^3] \\
&\quad + \frac{1}{8} [6 \mathcal{K}_1^{1D, \Sigma \neq 0} \tan^2 \theta' - (\cot \theta' + \tan \theta')^2 \mathcal{K}_3^{1D, \Sigma \neq 0} - (\cot \theta' + \tan \theta')^2 \mathcal{K}_4^{1D, \Sigma \neq 0} - \frac{3}{2} \mathcal{K}_4^{1D, \Sigma \neq 0} \tan^2 \theta' \\
&\quad - 3 \mathcal{K}_{13}^{1D, \Sigma \neq 0} \tan^2 \theta' + \frac{3}{2} \mathcal{K}_{14}^{1D, \Sigma \neq 0} \tan^2 \theta'] g_1^2 Z'^\mu Z'_\nu \text{tr}[X_\mu X^\nu] + [-\frac{1}{4} (\tan \theta' + \cot \theta')^2 - \frac{3}{8} \tan^2 \theta'] \mathcal{K}_1^{1D, \Sigma \neq 0} g_1^2 (\partial_\mu Z'^\mu)^2 \\
&\quad - \frac{3}{8} (\mathcal{K}_2^{1D, \Sigma \neq 0} + \mathcal{K}_{13}^{1D, \Sigma \neq 0}) (g_2^2 W^{\mu\nu a} W_{\mu\nu a} + g_1^2 B^{\mu\nu} B_{\mu\nu}) + \frac{3}{4} (\mathcal{K}_2^{1D, \Sigma \neq 0} - \mathcal{K}_{13}^{1D, \Sigma \neq 0}) g_1 \text{tr}[\overline{W}^{\mu\nu} \tau^3] (B_{\mu\nu} - \tan \theta' Z'_{\mu\nu}) \\
&\quad + \frac{3}{4} (\mathcal{K}_2^{1D, \Sigma \neq 0} + \mathcal{K}_{13}^{1D, \Sigma \neq 0}) \tan \theta' g_1^2 Z'_{\mu\nu} B^{\mu\nu} - \frac{1}{4} [\mathcal{K}_2^{1D, \Sigma \neq 0} (\tan \theta' + \cot \theta')^2 + \frac{3}{2} \mathcal{K}_2^{1D, \Sigma \neq 0} \tan^2 \theta' \\
&\quad + 9 \mathcal{K}_{13}^{1D, \Sigma \neq 0} (\tan \theta' + \cot \theta')^2 + \frac{3}{2} \mathcal{K}_{13}^{1D, \Sigma \neq 0} \tan^2 \theta'] g_1^2 Z'_{\mu\nu} Z'^{\mu\nu} + \frac{3i}{4} (\frac{1}{4} \mathcal{K}_3^{1D, \Sigma \neq 0} - \frac{1}{4} \mathcal{K}_4^{1D, \Sigma \neq 0} - \mathcal{K}_{13}^{1D, \Sigma \neq 0} \\
&\quad + \frac{1}{2} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) g_1 \tan \theta' Z'_\nu \text{tr}[X^\mu X_\mu] \text{tr}[X^\nu \tau^3] + \frac{1}{8} [-\frac{1}{2} (\cot \theta' + \tan \theta')^2 \mathcal{K}_3^{1D, \Sigma \neq 0} - \frac{3}{4} \tan^2 \theta' \mathcal{K}_3^{1D, \Sigma \neq 0} \\
&\quad + \frac{3}{4} \tan^2 \theta' \mathcal{K}_4^{1D, \Sigma \neq 0} - \frac{1}{2} (\cot \theta' + \tan \theta')^2 \mathcal{K}_4^{1D, \Sigma \neq 0} + 3 \tan^2 \theta' \mathcal{K}_{13}^{1D, \Sigma \neq 0} - \frac{3}{2} \tan^2 \theta' \mathcal{K}_{14}^{1D, \Sigma \neq 0}] g_1^2 Z'^2 \text{tr}[X^\mu X_\mu] \\
&\quad - \frac{3}{16} (\mathcal{K}_3^{1D, \Sigma \neq 0} + \mathcal{K}_4^{1D, \Sigma \neq 0}) i g_1^3 [\frac{1}{2} \tan^3 \theta' + (\cot \theta' + \tan \theta')^2 \tan \theta'] Z'_\mu Z'^2 \text{tr}[X^\mu \tau^3] + \frac{1}{64} (\mathcal{K}_3^{1D, \Sigma \neq 0} + \mathcal{K}_4^{1D, \Sigma \neq 0}) g_1^4 \\
&\quad \times [\frac{3}{2} \tan^4 \theta' + 6 \tan^2 \theta' (\cot \theta' + \tan \theta')^2 + (\cot \theta' + \tan \theta')^4] Z'^4 + \frac{3}{8} (\frac{1}{2} \mathcal{K}_4^{1D, \Sigma \neq 0} + \mathcal{K}_{13}^{1D, \Sigma \neq 0} - \frac{1}{2} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) \\
&\quad \times \text{tr}[X^\mu X_\nu] \text{tr}[X_\mu X^\nu] + \frac{3i}{4} (\frac{1}{2} \mathcal{K}_4^{1D, \Sigma \neq 0} + \mathcal{K}_{13}^{1D, \Sigma \neq 0} - \frac{1}{2} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) g_1 \tan \theta' Z'^\nu \text{tr}[X_\mu X_\nu] \text{tr}[X^\mu \tau^3] \\
&\quad + \frac{3}{16} (-\frac{1}{2} \mathcal{K}_4^{1D, \Sigma \neq 0} - \mathcal{K}_{13}^{1D, \Sigma \neq 0} + \frac{1}{2} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) g_1^2 \tan^2 \theta' Z'^2 \text{tr}[X_\mu \tau^3] \text{tr}[X^\mu \tau^3] + \frac{3i}{4} (-\mathcal{K}_{13}^{1D, \Sigma \neq 0} + \frac{1}{4} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) g_1 B_{\mu\nu} \\
&\quad \times \text{tr}[\tau^3 X^\mu X^\nu] + \frac{3i}{2} (-\mathcal{K}_{13}^{1D, \Sigma \neq 0} + \frac{1}{4} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) \text{tr}[X^\mu X^\nu \overline{W}_{\mu\nu}] + \frac{3}{4} (-\mathcal{K}_{13}^{1D, \Sigma \neq 0} + \frac{1}{4} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) g_1 \tan \theta' Z'_\mu \\
&\quad \times \text{tr}[\overline{W}^{\mu\nu} (X_\nu \tau^3 - \tau^3 X_\nu)] + \frac{3i}{4} (\mathcal{K}_{13}^{1D, \Sigma \neq 0} - \frac{1}{4} \mathcal{K}_{14}^{1D, \Sigma \neq 0}) g_1 \tan \theta' Z'_{\mu\nu} \text{tr}[X^\mu X^\nu \tau^3] \left. \right\}, \tag{A4}
\end{aligned}$$

$$\mathcal{K} = -\frac{1}{48\pi^2} \left( \log \frac{\kappa^2}{\Lambda^2} + \gamma \right) \quad \Lambda, \kappa: \text{ultraviolet and infrared cutoffs.} \tag{A5}$$

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